- 3. Sometimes the average may give a very absurd result. For example, if we are calculating average size of a family we may get a value 5.6. But this is impossible as persons cannot be in fractions. However, we should remember that it is an average value representing the entire group. For such situations one should report the approximate value.
- 4. Generally, a wrong choice of an average leads to fallacious and funny conclusions. So, a proper and judicious choice of an average for a particular problem is very important.
- 5. Measures of central value fail to give any idea about the formation of the series. Two or more series may have the same central value but may differ widely in compositions.

For example, let us consider the following series.

				Total	Mean
Series I	2	4	54	60	20
Series II	19	20	21	60	20
Series III	20	20	20	60	20

6. We must remember that an average is a measure of central tendency. Hence, unless the data shows a clear single concentration of observations, an average may not be meaningful at all. This evidently precludes the use of any average to a bimodal, a U-shaped or a J-shaped distribution.

Theoretical Questions

- What is a statistical average? What are the desirable properties for an average to possess?
- What is meant by various measures of central tendency? What purposes do their measurements serve?
- What is a weighted average? When should it be used instead of a simple average?
 - Define a 'weighted mean'. Under what circumstances would you prefer it to an unweighted mea
- What are the relative merits of the median and the mean as a measures of central tendency?
- What are 'measures of location'? In what circumstances would you consider them as the most suitable measures for describing the central tendency of a frequency distribution?
- Explain the properties of a good average. In the light of these properties which average do you think is the best and why?
- Define A.M, G.M. and H.M and also state the properties of A.M.
- "It is said that the choice of an average depends upon the particular problem in hand". Explain the statement.
- State the ideal properties for a good measure of central tendency.
- Explain the general limitations of an average.

Numerical Questions

- Find the arithmetic mean of the given data:
 - 25 50 15 10 45 30 20 40
- If the mean of the eight numbers 6, 4, 9, 10, 15, 5, 8, x is 10 then find the value of x.
- The mean of Supriya's marks on five tests was 77.4. The marks on the first 4 of these tests were 88, 77, 70 and 72. Find the mark on fifth test.
- The following is the frequency distribution of the number of telephone calls received in 245 successive one-minute intervals at an exchange:

No. of calls	0	1	2	3	4	5	6	7
Frequency	14	21	25	43	51	40	39	12

Obtain the mean number of calls per minute.

5. In an enquiry of 25 football matches, the number of goals scored recorded is as follows: 4, 0, 5, 2, 3, 1, 4, 3, 2, 3, 4, 3, 1, 6, 3, 2, 3, 4, 2, 1, 0, 3, 2, 5, 4

Represent the above data into discrete frequency distribution and then find the mean.

6. From the following data, find the mean.

Size	10	12	14	16	18	20
Frequency	3	7	12	20	8	5

7. The marks secured by BBS students in Public Youth Campus in Statistics are given below:

Marks	35	38	45	60	72	80	85
No. of Students	3	10	15		10	8	4

If the mean score of the marks distribution is 58, find out the missing number of students securing score 60 and the total number of students in the class.

8. The following data is related to the distance traveled by 520 villagers to buy their weekly requirements.

Miles traveled	2	4	6	8	10	12	14	16	18	20
No. of villagers	38	104	-	78	48	42	-	24	16	2

If mean is given to be 7.8, then find the missing frequencies.

Following data indicates the number of vehicles arrived during past 100 days in a certain tolling station.

Vehicles	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
No. of Days	3	14	53	20	10

Calculate average number of vehicles in a day.

10. The frequency distribution below represent the time in seconds needed to serve a sample of customers by cashiers at Bulls Eye Discount store in December 2010.

Time (seconds)	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99	100-109
Frequency	6	16	21	29	25	22	11	7	4

Calculate mean by (a) direct method and (b) coding method.

11. Calculate A.M. from the following data

Temp °C	-40 to -30	-30 to -20	-20 to -10	-10 to 0	0 to 10	10 to 20	20 to 30
No. of days	10	28	30	42	65	180	10

12. The following data provides the weekly wages of the worker in a firm. Calculate the average weekly wage per worker

Wages group (Rs.)	80-100	100-120	120-140	140-160	160-180	180-200
Total hours	168	170	225	272	126	91
Average no. of hours worked per worker	12	10	9	8.5	7	6.5

- 13. An investor buys Rs.1200 worth of share in company each month. During the first 5 months he bought the share at a price of Rs.10, 12, 15, 20 and 24 per share. After 5 months what is the average price paid for the shares by him.
- 14. Find the missing frequency from the following distribution of sales of shops, given that the mean sale of shops is Rs.24.625.

Sale ('00 Rs.)	0-10	10-20	20-30	30-40	40-50
No. of shops	5	25	-	18	7

15. Following are the monthly salaries in rupees of 32 employees of a firm.

91	139	126	119	100	87	65	77
116	76	69	88	112	118	89	116
65	77	99	95	108	127	86	148
97	105	95	80	86	106	93	135

The firm gives bonuses of Rs.10, 15, 20, 25, 30, 35, 40, 45 and 50 to employees in the respective salary groups: exceeding 60 but not exceeding 70, exceeding 70 but no exceeding 80 and so on up to exceeding 140 but not exceeding 150. Find out the total bonus paid per employee.

- 16. The mean weight of 150 students in a certain class is 60 kg. The mean weight of boys in the class is 70kg and that of girls is 55 kg. Find the number of boys and the number of girls in the class.
- 17. (a) The average income of 100 persons in village A is Rs. 3,500 and that of 150 persons in village B is Rs. 4,000. What is the total income of both the villages put together?



The mean marks got by 300 students in the subject of statistics are 45. The mean of the to 100 of them was found to be 70 and the mean of the last 100 was known to be 20. What the mean of the remaining 100 students?

- (c) The mean monthly salary paid to all employees in a certain company is Rs.600. The mean monthly salaries paid to the male and female were Rs.620 and Rs.520 respectively. Obtain the percentage of male and female employees in the company.
- (d) 100 students took a test. The result of those who secured less than 60% mark are given below:

Marks	0-20	20-40	40-60
No. of students	16	24	30

If the average mark of all students was 50, find out the average mark of those who secured more than 60% marks.

- 18. (a) There were 500 workers working in a factory. Their mean was calculated as Rs.200. Later on it was discovered that the wages of two workers were misread as 180 and 20 in place of 80 and 220. Find the correct average.
 - (b) The mean marks of 100 students were calculated as 65. At the time of checking it was found the marks 40 and 50 was wrongly copied and weeded out. Find the correct mean marks.
- 19. (a) Calculate the weighted mean price of a table from the following data, assuming that weights are proportional to the number of tables sold.

Price per table (Rs.)	36.00	40.00	44.00	48.00
No. of tables sold	14	11	9	6

- (b) In a certain course, a student's final mark is computed using the following weights: Quizzes 10%; Tests 40% and Examination 50%. Find his final average mark, if his marks in each of these categories 81, 75 and 70 respectively.
- (c) Dr. Khanal is very particular about the choice of pets. He selects kittens on the basis of 3 quotations: cuteness, fluffiness, and attitude. The quantities are given weights 5, 3, and x respectively. The scores for the three quantities in that order are 8, 9 and 4. If his weighted mean score was 7.5, what is the value of x, the weight for attitude?
- 20. (a) Six types of worker are employed in each of two workshops, but at different rates of wages as follows.

	Workshop	A	Work	shop B
Type of workers	Rate of wages per workers (Rs.)	No. of workers	Rate of wages	No. of workers
Mechanic	2.50	2	3.00	18
Fitter	3.50	14	3.00	50
Electrician	4.00	20	4.25	8
Carpenter	3.00	7	3.50	12
Smith	3.00	6	3.50	10
Clerk	2.00	1	5.00	2

In which of the two workshops is the average rate of wages per worker higher and by how much?



A contractor employs three types of workers- male, female and children. To a male worker, he pays Rs.16 per day, to a female worker Rs.13 per day and to a child worker Rs.10 per day. What is the average wage per day paid by the contractor if the number males, females and children is 20, 15 & 5 respectively?

- 21. Find the median of the following set of observations.
 - (a) 60, 70, 50, 80, 90, 100, 110
 - (b) 70, 80, 60, 90, 120, 140
- 22. In a batch of 15 students, 3 students failed in an examination. The marks of passed 12 students were 9, 6, 7, 8, 4, 5, 8, 10, 9, 7, 5, 7. What was the median mark of all 15 students?
- **23.** Following are the marks obtained by a batch of 10 students in a certain class tests in statistics (X) and accountancy (Y).

Roll No.	1	2	3	4	5	6	7	8	9	10
X	63	64	62	32	30	60	47	46	35	28
Y	68	66	35	42	26	85	44	80	33	72

In which subject is the level of knowledge of the students higher?

24. Find the median of the pop quiz marks shown below:

Marks	0	1	2	3	4	5
Frequency	1	1	5	3	2	1

25. The following is the income distribution of the persons:

Income (00 Rs.)	50-80	80-100	100-110	110-120	120-130	130-150	150-180	180-200
No. of persons	30	127	140	240	176	135	20	3

Find the median incomes.

26. Calculate the median annual income of a group of employees from the data given below:

Annual Income	Under	2000 -	3000 -	4000 -	5000 -	6000 -	7000 -	8000 -
in Rs.	2000	2999	3999	4999	5999	6999	7999	8999
No. of	15	32	65	79	90	57	36	14
Employees	0468	100000	120000	50000		2000.5	0.000	

27. Find the median wage of a labour from the following table

Wages (Rs.)	Above 0	Above 10	Above 20	Above 30	Above 40	Above 50	Above 60	Above 70
No. of Laborer	650	500	425	375	300	275	250	100

28. You are given below a certain statistical distribution:

Value	Less than 100	100- 200	200- 300	300- 400	400 & above	Total
No. of families	40	89	148	64	39	380

Calculate the most suitable average giving reason for your choice.

29. The expenditure of 1000 families is given as below:

Expenditure (Rs.)	40-59	60-79	80-99	100-119	120-139
No. of families	50	?	500	?	50

The median for the distribution is Rs.87. Calculate the missing frequencies.

30. The following table gives you the distribution of marks secured by some students in an examination.

Marks	0-20	21-30	31-40	41-50	51-60	61-70	71-80
No. of Students	42	38	120	84	48	36	31

Find (a) Median marks, (b) The percentage of failures if the minimum for a pass is 35 marks.

- 31. Five percent of the workers in a firm employing a total of 2000 men earn less than Rs.2.00 per hour, 450 earn from Rs.2 to 2.24 per hour, 35 percent earn from Rs.2.25 to Rs.2.49 per hour, 370 earn from Rs.2.50 to Rs.2.74 per hour, 12 percent earn from Rs.2.75 to Rs.2.99 per hour and the rest earn Rs.3 or more per hour. What is the median wage?
- 32. Calculate quartiles from the following data.

Months	1	2	3	4	5	6	7	8	9	10	11	12
Monthly	39	40	40	41	41	42	42	43	43	44	44	44
earnings	39	40	40	41	41	42	42	40	40	77	11	77



33. Calculate lower and upper quartiles from the following data.

Age in years	20	30	40	50	60	70	80
Number of members	3	61	132	153	140	51	3

34. Calculate the lower and upper quartiles from the following marks distribution:

Marks	Below 25	25-29	30-34	35-39	40-44	Above 44
Students	5	12	22	25	17	9

35. Find the (a) lower quartile, (b) upper quartile, (c) 7th decile, (d) 60th percentile for the following distribution.

Wages (Rs.)	30 -40	40 -50	50 -60	60 -70	70 -80	80 -90	90 -100
No. of persons	1	3	11	21	43	32	9

36. In a small branch of a bank in a rural area, the following is the average deposit balance of current accounts during a month. Calculate median, seventh decile and 85th percentile.

Deposit balance (less than)	1000	900	800	600	550	500	400	250	100
No. of deposit	500	498	480	475	440	374	300	125	25

37. Calculate the median, 3rd decile and 20th percentile from the following data.

Central size	2.5	7.5	12.5	17.5	22.5
Frequency	7	18	25	30	20

Calculate the median income from the following income distribution. Also find out the locest income of highest 10% earners.

Income in Rs. 000 (Less than)	5	10	15	20	25	34	35	40
No. of Earners	5	25	60	100	125	140	150	156

39. The monthly salary distribution of 250 families in a certain locality is given below.

Monthly salary (Rs.)	No. of families	Monthly salary (Rs.)	No. of families
More than 0	250	More than 2000	55
More than 500	200	More than 2500	30
More than 1000	120	More than 3000	15
More than 1500	80	More than 3500	5

Find the following:

- (a) Limits of the income of middle 50% of the families and
- (b) If income tax is to be levied on families whose income exceeds Rs. 1,800 per month, calculate the percentage of families, which will be paying income tax.
- 40. One hundred and twenty students appeared for a certain test and the following marks distribution was obtained.

Marks	0 -20	20 -40	40 -60	60 -80	80 -100
Students	10	30	36	30	14

Find:

- (a) The limits of marks of middle 30% students.
- (b) The percentage of students getting marks more than 75.
- (c) The number of students who fail, if 35 marks are required for passing.
- 41. (a) When 11 students were asked for their favourite number, the responses were: 7, 10, 72, 7, 1600, 4, 1, 7, 2, 1, 7.

What measure of central tendency is the most appropriate to describe their answers?

(b) Compute mode from the following:

x	50	100	150	200	250	300	350	400
f	5	14	40	91	150	87	60	38

42. Calculate the modal size in the following distribution:

Size (inches)	Below 10	10-12	12-14	14-16	16-18	18-20
Demand	3	15	27	20	3	2

Will the median fall under the same size?

43. Calculate the modal value from the following data:

Income (Rs.) (less than)	100	200	300	400	500	600
No. of persons	8	22	35	60	67	70

44. Determine the mode using empirical relation between mean, median and mode.



x	20	21	22	23	24	25	26	27	28	29
f	6	9	4	2	10	8	7	5	1	3

45. Calculate the mode for the following data by using empirical relationship.

Annual Wages up to Rs.	1200	1300						
No. of worker	12	30	80	100	150	220	300	330

46. The following table represents the marks of 100 students.

Marks	0-20	20-40	40-60	60-80	80-100
No. of students	14	-	27	-	15

If the mode value is 48 find the missing frequencies and the mean marks of all 100 students.

47. You are given the following incomplete frequency distribution. It is known that the total frequency is 1000 and that the median is 413.11. Estimate by calculation, the missing frequencies and find the value of the mode.

W-1 - (-)	300-	325-	350-	375-	400-	425-	450-	475-
Value (x)	325	350	375	400	425	450	475	500
Frequency (f)	5	17	80	?	326	?	88	9

48. Calculate the most repeated value from the following data.

Marks	0 - 10	10 - 20	20 - 40	40 - 50	50 - 70
No. of students	5	15	40	32	28

49. Calculate mean, median and mode from the following data of the heights in inches of a group of students.

61 62 63 61 61 64 64 60 65 63 64 65 66 64	61	62	63	61	61	64	64	60	65	63	64	65	66	64
---	----	----	----	----	----	----	----	----	----	----	----	----	----	----

Now, suppose that a group of students whose heights are 60, 66, 59, 68, 67 and 70 inches is added to the original group. Find the mean, median and mode of the combined group.

50. A locality with 16 schools has the following distribution of average number of teachers in different income groups.

Income (central value)	600	800	1000	1200	1400
No. of schools	2	3	5	4	2
Average no. of teachers	20	26	21	21	9

Find the mean, median and modal income of all the teachers.

51. A cement company sells his production in different cities through the appointed dealers. The sales of his production in the last year is given in the following tabular form:

Sales (in '00' bags)	0 - 500	500 - 1000	1000 - 1500	1500 - 2000	2000 - 2500	2500 and more
Number of dealers	40	48	60	52	35	22

The annual general meeting of the company decided to give award of Rs. 5,000 to each dealer whose sales are more than the most usual sales. Calculate the total amount of money that would be given to the dealers.

- 2. (a) For moderately asymmetrical distribution, the arithmetic mean = 28 and the median = Find mode of this distribution.
 - (b) In a moderately asymmetrical distribution, the values of mode and mean are 32.1 and 35.4 respectively. Find the median value.
- 53. (a) Calculate G.M. and H.M. of the following data.

i)	Variable val	ue (X)	5	7	9	12	18
ii)	X	10		20	30	40	55
	f	2		4	8	3	2
ii)	C.I.	5-14	15	5-24	25-34	35-44	45-54
	f	5		7	10	4	2

- (b) A cyclist pedals from his house to his college at a speed of 10km per hour and back from the college to his house at 15km per hour. Find the average speed.
- (c) You make a trip which entrails traveling 900km by train at a speed of 60km/hour, 3000km by boat at an average of 25km/hour, 400km by plane at 350 km/hour and finally 15km by taxi at 25km/hour, what is your average speed for the entire distance?
- (d) If H.M., A.M., and G.M. of a set of 5 observations are 10.2, 16 & 14 respectively. Comment upon these values.

Additional Problems

The operation manager of a digital watch company is considering to switch from batch production process to continuous assembly line. He conducted a time study of the batch process. From the data he collected, the following frequency distribution of total production time for one watch resulted.

Production time (in minutes)	Below 5	5.01 - 5.50	5.51 - 6.00	6.01 - 6.50	6.51 - 7.00	Above 7.00
Frequency	16	22	39	21	19	14

If the median is over 6.00 minutes, operation manager can switch from batch production process to continuous assembly line. What should be his decision?

The following are the weekly production in units of 60 workers in a manufacturing company.

23	35	41	82	75	46	48	88	72	44	85	51
51	77	62	75	68	50	64	25	49	56	55	16
82	39	32	51	52	62	19	52	54	63	45	56
33	48	67	59	40	54	50	64	46	69	57	40
39	49	55	53	20	55	72	57	52	57	42	75

The manager has decided to give bonus of Rs.5, 10, 15, 20 and 25 to each worker in the respective output group of 40 or over weekly output. Find the average bonus received by the workers.

3 A factory pays worker on piece rate basis and also a bonus to each worker on the basis of individual output in each quarter. The rate of bonus payable is as follows.

Output in units	70-74	75-79	80-84	85-89	90-94	95-99	100-104
Bonus in rupees	40	45	50	60	70	80	100

The individual output of a batch of 50 workers is given below

94	83	78	76	88	86	93	80	91	82
89	87	84	85	97	88	97	83	92	88
80	75	84	81	75	72	82	95	93	83
80	86	101	98	85	99	82	77	83	81
82	87	98	87	89	71	103	90	72	80

By suitable classification, you are required to find:

- (a) Average bonus per worker for the quarter
- (b) Total quarterly bonus paid to the whole batch
- (c) Average output per worker
- 4. Incomes of employees in an industrial concern are given below. The total income of the 10 employees in the class over Rs. 250 is Rs. 3000. Compute the mean income. Every employee belonging to the top 25% of the earners is required to pay 1% of his income ot workers relief fund. Estimate the contributions to this fund.

Income (Rs.)	Frequency	Income (Rs.)	Frequency
0 -50	90	150 -200	80
50 -100	150	200 -250	70
100 -150	100	250 and over	10

Answers

1. 27.5

2. 23

3. 80

4. 3.763

5. 2.8

- 6. 15.38
- 7. f₁(missing freq.) = 20; Total freq. = 70 8. 138 and 30
- 9. 27 vehicles

10. (a) 60.10 (b) 60.10

11. 4.29°C

12. Rs. 140.83

13. Rs. 14.63

14. 25

15. Rs. 26.56

- 16. Boys = 50 and Girls = 100
- 17. (a) Rs. 950,000 (b) 45 (b) 80%, 20% (c) 87.33

18. (a) 200.2 (b) 65.41

- 19. (a) Rs.40.70 (b) $\bar{x}_{w} = 73.1$ (c) x = 2
- 20. (a) $\bar{X}_A = Rs.3.50$; $\bar{X}_B = Rs.3.25$, In workshop A, by 25 paisa (b) Rs. 14.13
- 21. (a) 80 (b) 85

- 22. 7 marks
- 23. Ma(x) = 46.5, Ma(y) = 55, level of knowledge of students is higher in Accountancy

- 25. Rs. 11577.08
- 26. Ma = Rs.5032.83

27. M_d = Rs.36.67

- 28. M_d = Rs.241.22
- 29. 262.5 ≈ 263, 137

- 30. M_d = 40.46, 33.58%
- 31. Rs. 2.41

32. Q1 = 40.25 and Q3 = 43.75

- 33. $Q_1 = 40 \text{ yrs}$ and $Q_3 = 60 \text{ yrs}$.
- 34. Q1 = 30.75 marks and Q3 = 40.53 marks
- 35. (a) Rs. 67.14, (b) Rs. 83.44, (c) Rs. 81.56, (d) Rs. 78.37
- 36. $M_d = 357.14$, $D_7 = 467.57$, $P_{85} = 538.64$ 37. $M_d = 15$, $D_3 = 11$ and $P_{20} = 8.61$ 38. M_d = Rs. 17250; P₉₀ = Rs. 30,200
 - 39. (a) Q1 = Rs. 578 (approx.), Q3 = Rs.1850, (b) 26%
- 40. (a) $P_{35} = 41.11$, $P_{65} = 61.33$; (b) 17.9%; (c) $32.5 \approx 33$
- 41. (a) Mode = 7 (b) Mode 250
- 42. Mode = 13.26 inches; M_d = 13.26 inches, Yes the median falls under the same size
- 43. Mode = Rs.340

44. 24.18

45. Rs. 1704.89

- 16. $f_1 = 23$, $f_2 = 21$, $\kappa = 50$
- 47. 227 and 248, Mode = 413.98
- 48. 44 marks

First group: Mean = 63.07, M_d = 63.5, Mode = 64;

Combined group: Mean = 63.65, Md = 64, Mode = 64

- 50. Mean = Rs. 976.62, Md = Rs. 984.76, Mode = Rs. 1012.50
- 51. Rs. 6,65,000
- 52. (a) Mode = 19 (b) 34.3
- 53. (a) (i) 9.259, (ii) 27.369, (iii) 23.506 (b) 12km/hr, (c) 31.56 km/hr.

Additional Problems

- 1. Md = 5.86 minutes; Since the median is not over 6.00 minutes the operation manager cannot switch from batch production to continuous assembly line. Hence, the manager continuous batch production process.
- 2. Rs. 12
- 3. (a) Rs.59.3; (b) Rs.2965; (c) 85.9 units
- 4. Mean income = Rs. 117.50; estimated contribution to the fund = Rs. 271.20

